Int. J. Heat Mass Transfer. Vol. 26, No. 2, p. 316, 1983
Printed in Great Britain

0017-9310/83/020316-01 $03.00/0
Pergamon Press Ltd.

SOME PROPERTIES OF THE COEFFICIENT MATRIX OF THE DIFFERENTIAL EQUATIONS FOR PARALLEL-FLOW MULTICHANNEL HEAT EXCHANGERS

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(*Received* 19 *January* 1982)

NOMENCLATURE

- $\bar{A}$, coefficient matrix;
- $g(\lambda)$, characteristic equation of matrix $\bar{A}$;
- k_{ij}, overall length conductance for heat transfer between channels i and j;
- λ_i, eigenvalue of matrix $\bar{A}$;
- $\bar{t}$, temperature column vector;
- W_i, heat capacity rate of the fluid in channel i;
- x, space coordinate along the stream;
- y_i, unknown quantity;
- $|y_i|$, amplitude of y_i.

THE SYSTEM of equations for parallel-flow multichannel exchangers written in the matrix form is

$$\bar{A}\bar{t} = \frac{d\bar{t}}{dx} \tag{1}$$

where $\bar{A}$ is the $n \times n$ coefficient matrix

$$\bar{A} = \begin{bmatrix} \frac{K_1}{W_1} & \frac{k_{12}}{W_1} & \cdots & \frac{k_{1n}}{W_1} \\ \frac{k_{21}}{W_2} & \frac{K_2}{W_2} & \cdots & \frac{k_{2n}}{W_2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{k_{n1}}{W_n} & \frac{k_{n2}}{W_n} & \cdots & \frac{K_n}{W_n} \end{bmatrix} \tag{2}$$

and

$$K_j = -\sum_{\substack{i=1 \\ i \neq j}}^{n} k_{ji}; \; i, j = 1, 2, \ldots, n. \tag{3}$$

Wolf [1] proves that all the eigenvalues of matrix $\bar{A}$ are distinct. His proof is based on an invalid assumption that two polynominals of the same power n with different coefficients cannot have equal roots. It is obvious that they can have even $n-1$ equal roots.

The general solution of the equation (1) presented by Wolf, even together with a complement given in ref. [2], is not a general solution at all, for multiple non-zero eigenvalues of matrix $\bar{A}$ may occur.

Example

Data.

$n = 3$

$k_{12} = 0.8$; $k_{13} = 0.64$, $k_{23} = 1.28$ W m^{-1} K^{-1};

$W_1 = 25$, $W_2 = 50$, $W_3 = 40$ W K^{-1}.

Characteristic equation of matrix $\bar{A}$.

$$g(\lambda) = -\lambda(-\lambda - 0.0736)^2 = 0$$

Roots. $\lambda_1 = 0$, $\lambda_2 = \lambda_3 = -0.0736\,\text{m}^{-1}$

It has been proved by Settari that all the eigenvalues of matrix $\bar{A}$ are real [3]. The proof is incorrect because matrix $\bar{B}$ (ref. [3] p. 556) should be non-negative definite (ref. [4], p. 156) but it is not so, for the fluid heat capacities W_i ($i = 1, 2, \ldots, n$) can be both negative and positive. It will be now proved that all the eigenvalues of matrix $\bar{A}$ are real.

Let us suppose that the determinant of matrix A is the characteristic determinant of the following system of linear algebraic equations

$$\begin{aligned} &\left(\frac{K_1}{W_1} - \lambda_k\right)y_1 + \frac{k_{12}}{W_1}y_2 + \ldots + \frac{k_{1n}}{W_1}y_n = 0 \\ &\frac{k_{21}}{W_2}y_1 + \left(\frac{K_2}{W_2} - \lambda_k\right)y_2 + \ldots + \frac{k_{2n}}{W_2}y_n = 0 \\ &\frac{k_{n1}}{W_n}y_n + \frac{k_{n2}}{W_n}y_2 + \ldots + \left(\frac{K_n}{W_n} - \lambda_k\right)y_n = 0 \end{aligned} \tag{4}$$

where λ_k is an eigenvalue of matrix $\bar{A}$. We do not know if λ_k is real or complex yet.

For det $\bar{A} = 0$, the set of equations (4) must have at least one non-zero solution. Let us transform the system (4) in the following manner: (1) Multiply the first equation by W_1, the second one by W_2, etc. (2) Transpose expressions $W_i y_i \lambda_k$ to the RHS of the equations.

(3) Multiply the first equation by $\bar{y}_1$ (the number conjugate with y_1), the second equation by $\bar{y}_2$, etc.

(4) Sum separately the LHS and RHS of the equations. Thus the following equation results:

$$\begin{aligned} &K_1|y_1|^2 + K_2|y_2|^2 + \ldots + K_n|y_n|^2 + k_{12}(y_1\bar{y}_2 + y_2\bar{y}_1) \\ &+ \ldots + k_{ij}(y_i\bar{y}_j + y_j\bar{y}_i) + \ldots + k_{n-1,n}(y_{n-1}\bar{y}_n + y_n\bar{y}_{n-1}) \\ &= \lambda_k(W_1|y_1|^2 + W_2|y_2|^2 + \ldots + W_n|y_n|^2). \end{aligned} \tag{5}$$

The number $y_i\bar{y}_j$ is conjugate with $y_j\bar{y}_i$, therefore the numbers $y_i\bar{y}_j + y_j\bar{y}_i$ are real. Each eigenvalue of the matrix $\bar{A}$ is a quotient of real numbers. This implies that all the eigenvalues of $\bar{A}$ are real.

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